11/10/2015

سكشن [٣] رياضة

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★ Find solution or Find value or Find roots.

[1] $(-2-2\sqrt{3}\,i)^{1/4} = Z^{1/4}$

$$Z_K^{1/n} = r^{1/n} e^{i\left(\frac{\theta+2K\pi}{n}\right)}$$

$$Z_K^{1/n} = r^{1/n}\left[\cos\left(\frac{\theta+2K\pi}{n}\right)+i\sin\left(\frac{\theta+2K\pi}{n}\right)\right]$$

$\rightarrow r = |z| = \sqrt{4+4\times3} = 4$

$\theta = \arg(z) = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = \pi + \frac{1}{3}\pi = \frac{4}{3}\pi$.

ربع ثالث

$\therefore Z^{1/4} = (4)^{1/4}\cdot\left[\cos\left(\frac{\frac{4\pi}{3}+2K\pi}{4}\right)+i\sin\left(\frac{\frac{4\pi}{3}+2K\pi}{4}\right)\right]$

∴ الجذور

at $n=0 \longrightarrow Z_0^{1/4} = \frac{\sqrt{2}}{2}+\frac{\sqrt{6}}{2}i$

$n=1 \longrightarrow Z_1^{1/4} = -\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}i$

$n=2 \longrightarrow Z_2^{1/4} = \frac{-\sqrt{2}}{2}-\frac{\sqrt{6}}{2}i$

$n=3 \longrightarrow Z_3^{1/4} = \frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}i$

★ Find solution $Z^{3/2} = 4\sqrt{2}+i\,4\sqrt{2}$

(بنفس القانون $Z_K^{1/n}$)

$Z = (4\sqrt{2}+i\,4\sqrt{2})^{2/3} = r^{2/3}\left[\cos\left(\frac{2\theta+4K\pi}{3}\right)+i\sin\left(\frac{2\theta+4K\pi}{3}\right)\right]$

$r = \sqrt{(4\sqrt{2})^2+(4\sqrt{2})^2} = 8$.

$\theta = \tan^{-1}(1) = \pi/4$.

$= (8)^{2/3}\left[\cos\left(\frac{2\pi/4+4K\pi}{3}\right)+i\sin\left(\frac{2\pi/4+4K\pi}{3}\right)\right]$

الجذور ∴

$Z_0^{2/3} =$

$Z_1^{2/3} =$

$Z_2^{2/3} =$

b ★ **Find roots** $z^3 - 1 = 0$

$$z^3 = 1 \longrightarrow z = (1)^{1/3} = (1 + 0i)^{1/3}$$

$$= r^{1/3}\left[\cos\left(\frac{\theta + 2k\pi}{3}\right) + i\sin\left(\frac{\theta + 2k\pi}{3}\right)\right]$$

$\longrightarrow r = 1$

$\longrightarrow \theta = \arg(z) = \tan^{-1}(0/1) = 0$

الجذور

$$z_0^{1/3} = (1)^{1/3}[\cos 0 + i\sin 0] = 1 + 0i.$$

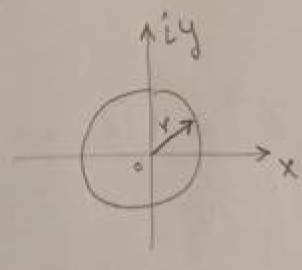
$$z_1^{1/3} = (1)^{1/3}\left[\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right] = \frac{\sqrt{3}}{2} + \frac{1}{2}i.$$

$$z_2^{1/3} = (1)^{1/3}\left[\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)\right] = \frac{\sqrt{3}}{2} - \frac{1}{2}i.$$

★ Some curves in complex plane.

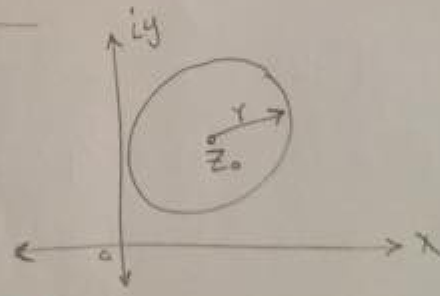
1 $|z| = r$

دايرة نصف قطرها r
ومركزها (0,0)



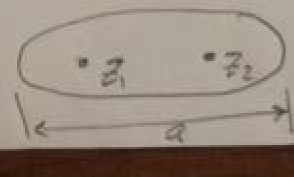
2 $|z - z_0| = r$

دائرة مركزها z_0
ونصف قطرها r



3 $|z - z_1| + |z - z_2| = a$

قطع ناقص



④ $|z - z_1| - |z - z_2| = a$

قطع زائد .



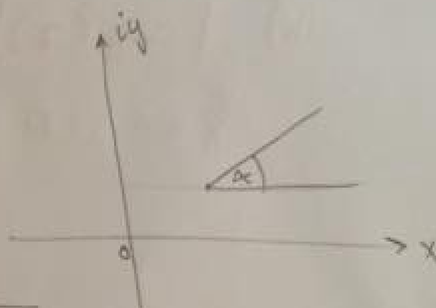
⑤ $\arg(z) = \alpha$

جميع النقاط الواقعة على الخط المستقيم الذى يصنع زاوية α مع محور x ونقطة الأصل .



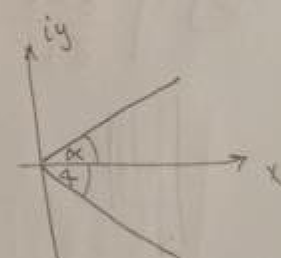
⑥ $\arg(z - z_0) = \alpha$

جميع النقاط الواقعة على الخط المستقيم الذى يصنع زاوية α مع الموازى لمحور x ونقطة z_0



⑦ $|\arg(z)| \leqslant \alpha$

$-\alpha \leqslant \arg \leqslant \alpha$



هام : ① فى حالة > أو < نرسم المنحنى فقط فى حالة التساوى

② المعادلة ليست كالسابق عوض عن $z = x + iy$

وشوف المعادلة الناتجة تمثل ايه ؟

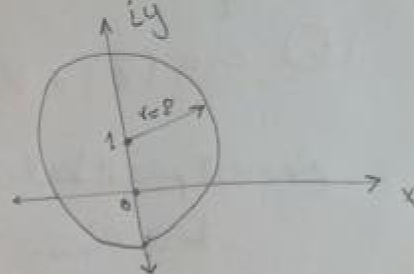
d

★ Describe & graph.

a $|z - i| = 2$

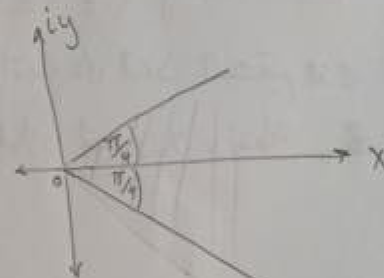
$\to (0 + 1i)$

دايرة مركزها $(0,i)$ ونصف قطرها 2.



b $|\arg(z)| \leq \pi/4$

$-\pi/4 \leq \arg(z) \leq \pi/4$



c $\left|\frac{z-i}{z+i}\right| = 1$

$$\left|\frac{x+iy-i}{x+iy+i}\right| = 1 \longrightarrow \left|\frac{x+i(y-1)}{x+i(y+1)}\right| = 1.$$

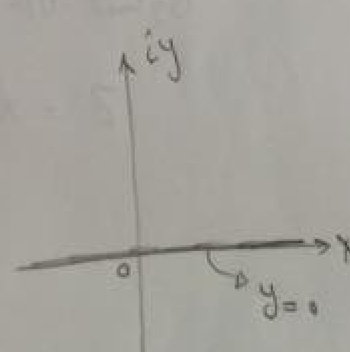
$$\frac{|x+i(y-1)|}{|x+i(y+1)|} = \frac{\sqrt{x^2+(y-1)^2}}{\sqrt{x^2+(y+1)^2}} = 1$$

$$\sqrt{x^2+(y-1)^2} = \sqrt{x^2+(y+1)^2}$$

$$\cancel{x^2} + (y-1)^2 = \cancel{x^2} + (y+1)^2$$

$$y^2 - 2y + 1 = y^2 + 2y + 1$$

$$-y = y \Longrightarrow -2y = 0 \longrightarrow \boxed{y = 0} \#$$

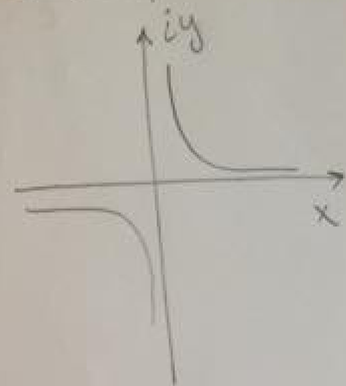


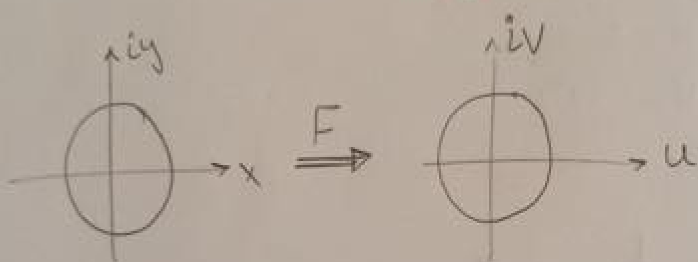
$$Im(z^2) = 4$$

$$Im((x+iy)^2) = 4 \quad = Im(x^2 + i2xy - y^2) = 4$$

$$2xy = 4$$

$$xy = 2 \longrightarrow \boxed{y = 2/x}$$





Z plane

w plane

* write the Following Function on the Form

$$w = u + iV.$$

$$F(z) = u + iV$$

يقسم الدالة لجزء حقيقي وجزء
تخيلي

نستخدم $Z = x + iy$

ماعدا $Z^{\text{رقم} > 2}$

$Ln(Z)$

نستخدم $Z = re^{i\theta}$

F

Ex.- $F(z) = z^2$ z plan ⟶ w plan.

$$z = (x+iy)$$

$$F(z) = (x+iy)^2 = x^2 + i2xy - y^2$$

$$F(z) = (x^2 - y^2) + i(2xy)$$

$$F(z) = u + iv$$

$$u = x^2 - y^2$$

$$v = 2xy$$

Ex $F(z) = \frac{1}{z} + z$

$$F(z) = \frac{1+z^2}{z} = \frac{1+(x+iy)^2}{(x+iy)} = \frac{1+x^2+i2xy-y^2}{x+iy}$$

نضرب بالمرافق $* \frac{x-iy}{x-iy}$

$$F(z) = \frac{x + x^3 + i2x^2y - xy^2 - iy - ixy^2 + iy^3}{x^2+y^2}$$

$$F(z) = \frac{x + x^3 + ix^2y + xy^2 - iy + iy^3}{x^2+y^2}$$

$$F(z) = \underbrace{\frac{x+x^3+xy^2}{x^2+y^2}}_{u} + i\,\underbrace{\frac{x^2-y+y^3}{x^2+y^2}}_{v}$$

3 $F(z) = z e^{2z}$

$$F(z) = (x+iy) \cdot e^{2(x+iy)} = (x+iy) \cdot e^{(2x+i2y)}$$

$$= (x+iy) \cdot e^{2x} \cdot e^{i2y}$$

$$= (x+iy) \cdot e^{2x} \cdot \left[\cos(2y) + i \sin(2y)\right]$$

$$= (xe^{2x} + iye^{2x})\left[\cos(2y) + i \sin(2y)\right]$$

$$F(z) = \left[xe^{2x}\cos(2y) + i\, xe^{2x}\sin(2y) + i\, ye^{2x}\cos(2y) - ye^{2x}\sin(2y)\right]$$

$$\therefore\ u = xe^{2x}\cos(2y) - ye^{2x}\sin(2y).$$

$$v = xe^{2x}\sin(2y) + ye^{2x}\cos(2y).$$

4 $F(z) = \ln(z)$

$$F(z) = \ln(re^{i\theta}) = \ln r + \ln e^{i\theta}$$

$$= \ln r + i\theta \longrightarrow \text{principle value}$$

$$= \ln r + i(\theta + 2n\pi)$$

$\rightarrow$ multi - value.